B.Tech. EAS-201

Second Semester Examination 2009-10

Engineering Physics-II

Note : (i) Attempt all questions.
(ii) Marks of each question are shown against it.
(iii) The physical constants are given at the end of the question paper.

SECTION-A

Q. 1. Attempt all parts. All parts carry equal marks. Pick the correct choice from the following : **(1 × 10 = 10)**

Q. 1. (a) The masses of neutron and electron are m_n and m_e respectively. If they have the same de Broglie wavelength, then their velocities should be in the ratio.

(i) 1 : 1 **(ii) $\frac{m_e}{m_n}$**

(iii) $\frac{m_n}{m_e}$ **(iv) $\frac{m_e^2}{m_n^2}$**

Ans. (ii) $\frac{m_e}{m_n}$

Q. 1. (b) The characteristics of wavefunction ψ are :

(i) Real function, finite and discontinuous.
(ii) Complex single valued, finite and continuous function.
(iii) Complex infinite and discontinuous function.
(iv) Complex single valued and infinite.

Ans. (ii) Complex single valued, finite and continuous function.

Q. 1. (c) Compton shift is given by :

(i) $\frac{h}{m_0c}(1 - \sin\phi)$ **(ii) $\frac{h}{m_0c^2}(1 - \cos\phi)$**

(iii) $\frac{h}{m_0c}(1 - \cos\phi)$ **(iv) None of these**

Ans. (iii) $\frac{h}{m_0c}(1 - \cos\phi)$

Q. 1. (d) Which of the following is not equal to the dielectric constant.

(i) $\frac{V_0}{V}$ **(ii) $\frac{C_0}{V}$**

(iii) $\frac{E_0}{E}$ **(iv) $\frac{\epsilon_0}{\epsilon}$**

Ans. (ii) $\frac{C_0}{V}$ or (iv) $\frac{\epsilon_0}{\epsilon}$

Q. 1. (e) In an isotropic dielectric medium.

(i) D and E are pendicular to each other
(ii) D and E are inclined at 45° to each other
(iii) D and E have the same direction

(iv) D and P are perpendicular to each other

Ans. (iii) D and E have the same direction

Q. 1. (f) Ferromagnetic substances have.

(i) High permeability and low susceptibility

(ii) Low permeability and high susceptibility

(iii) Both permeability and susceptibility low

(iv) Both permeability and susceptibility high

Ans. (iv) Both permeability and susceptibility high

Q. 1. (g) Which of the following equipments are used for the production of ultrasonic waves ?

(i) Piezoelectric generator **(ii) magnetostriction oscillator**

(iii) both (i) and (ii) **(iv) None of these**

Ans. (iii) Both (i) and (ii)

Q. 1. (h) The expression for displacement current density is.

(i) $Jd = \epsilon_0 \dfrac{\partial E}{\partial t}$ (ii) $Jd = \dfrac{1}{\epsilon_0} \dfrac{\partial E}{\partial t}$

(iii) $Jd = \dfrac{\partial E}{\partial t}$ (iv) $Jd = \dfrac{1}{4\pi \epsilon_0} \dfrac{\partial E}{\partial t}$

Ans. (i) $Jd = \epsilon_0 \dfrac{\partial E}{\partial t}$

Q. 1. (*i*)A super conducting material exhibits.

(i) Zero resistivity and complete diamagnetism

(ii) Zero conductivity and complete diamagnetism

(iii) Zero resistivity and complete paramagnetism

(iv) Infinite conductivity and complete permagnetism

Ans. (i) Zero resistivity and complete diamagnetism

Q. 1. (j) At nanoscale the surface to volume ratio is :

(i) very low **(ii) very high**

(iii) equal to one **(iv) equal to five**

Ans. (ii) very high

SECTION-B

Q. 2. Attempt any three parts. All parts carry equal marks. **$(3 \times 5 = 15)$**

Q. 2. (a) Calculate the deBroglie wavelength of neutron of energy 12.8 MeV.

Ans. Given : Energy $E = 12.8 \times 10^6$ eV $= 12.8 \times 10^6 \times 1.6 \times 10^{-19}$

$= 20.48 \times 10^{-13}$ Joule

de-Broglie wavelength, $\lambda = ?$

We know that de-Broglie wavelength of neutron

$$\lambda_n = \frac{h}{\sqrt{2m_n E}} = \frac{6.623 \times 10^{-34}}{\sqrt{2 \times (1.67 \times 10^{-27}) \times (20.48 \times 10^{-13})}}$$

$= 0.801 \times 10^{-14}$ meter $= 8.01 \times 10^{-15}$ meter

$= 8.01 \times 10^{-5}$ Å

Q. 2. (b) A nucleon is confined to a nucleus of diameter 5×10^{-14} m. Calculate minimum uncertainty in the momentum of the nucleon. Also calculate the minimum kinetic energy of the nucleon.

Ans. Given : Diameter of nucleus, $D = 2r = 5 \times 10^{-14}$ m$=(\Delta x)_{max}$

$$(\Delta p)_{min} = ?, E_{min} = ?$$

From Heisenberg uncertainty principle, we know that

$$\Delta x \cdot \Delta p = \frac{h}{4\pi}$$

$$\text{or } (\Delta p_{min}) = \frac{h}{4\pi \times (\Delta x)_{max}} = \frac{6.62 \times 10^{-34}}{4 \times \frac{22}{7} \times 5 \times 10^{-14}}$$

$$= 1.054 \times 10^{-21} \text{ kg-m/sec}$$

$$E_{min} = \frac{p^2}{2m} = \frac{(1.054 \times 10^{-21})^2}{2 \times 1.67 \times 10^{-27}} = 3.32 \times 10^{-16} \text{ Joules}$$

$$= \frac{3.32 \times 10^{-16}}{1.6 \times 10^{-19}} = 2.075 \times 10^3 \text{ eV}$$

Q. 2. (c) A particle is in motion along a line between $x = 0$ and $x = a$ with zero potential energy. At points for which $x < 0$ and $x > a$, the potential energy is infinite. The wavefunction for the particle in nth state is given by :

$$\psi_n = A \sin \frac{n\pi x}{a}$$

Find the expression for the normalised wavefunction.

Ans. For normalised wavefunction, we know that

$$\int_{\infty}^{\infty} |\psi_n(x)|^2 \, dx = 1$$

$$\text{or} \quad \int_{\infty}^{0} 0 \cdot dx + \int_0^a A \cdot \sin^2 \frac{n\pi x}{a} \, dx + \int_2^{\infty} 0 \, dx = 1$$

$$\text{or} \quad \frac{A^2}{2} \int_0^a 2 \ln^2 \frac{n\pi x}{a} \, dx = 1 \quad \text{or} \quad \frac{A^2}{2} \int_0^a \left[1 - \cos \frac{2n\pi x}{a}\right] dx = 1$$

$$\text{or} \quad \frac{A^2}{2} \left[x - \frac{\sin \frac{2n\pi x}{a}}{\frac{2n\pi}{a}}\right]_0^a = 1 \quad \text{or} \quad \frac{A^2}{2}(a - 0) = 1$$

$$\text{or} \quad A^2 = \frac{2}{a} \Rightarrow A = \sqrt{\frac{2}{a}}$$

$\therefore$ Normalised wavefunction is, $\psi = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$

Q. 2. (d) A quartz crystal of thickness 0.001 m is vibrating at resonance. Calculate the fundamental frequency.

Given : Young modulus for quartz is 7.9×10^{10} Nm^{-2} and density of quartz is 2.65×10^3 kg/m^3.

Ans. Given : Thickness of crystal, $t = 0.001$ m, $y = 7.9 \times 10^{10}$ N/m^2, $\rho = 2.65 \times 10^3$ kg/m^3, $\nu = ?$

We know that, $\nu = \dfrac{1}{2t}\sqrt{\dfrac{y}{\rho}} = \dfrac{1}{2 \times 0.001} \times \sqrt{\dfrac{7.9 \times 10^{10}}{2.65 \times 10^3}} = 2.725 \times 10^6 \text{ Hz} = 2.725 \text{ MHz}$

Q. 1. (e) Assuming that all the energy from a 1000 watt lamp is radiated uniformly. Calculate the average values of the intensities of electric and magnetic fields of radiation at a distance of 2 m from the lamp.

Ans. Given : Energy, $E = 1000$ watt, $r = 2$ m.

We know that $\quad s = \dfrac{E}{4\pi z^2} = \dfrac{1000}{4 \times 3.149 \times 2^2} = 19.9 \text{ watt/m}^2$

Also we know that, $\quad s = E \times H = 19.9 \text{ watt/m}^2$ ...(1)

and $\quad \dfrac{E}{H} = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = 37.7\ \Omega$ ...(2)

Multiplying (1) and (2) side by side

We have, $E^2 = 19.9 \times 377$

$\therefore \quad E = \sqrt{19.9 \times 377} = 86.6$ V/m

Dividing (1) by (2), we have $H^2 = \dfrac{19.9}{377}$

$\therefore \quad H = \sqrt{\dfrac{19.9}{377}} = 0.2297$ amp turn/m

SECTION-C

Note : *Attempt all questions. All questions carry equal marks.* $(5 \times 5 = 25)$

Q. 3. Attempt any one part of the following :

Q. 3. (a) What is uncertainty principle ? Apply this to calculate the radius of the Bohr's first orbit.

Ans. Heisenberg's Uncertainty Principle : The Heisenberg's uncertainty principle or the principle of indeterminancy was proposed by Werner Heisenberg in 1927. According to which "it is impossible to measure precisely and simultaneously both the members of pairs of certain canonically conjugate variables that describe the behaviour of an atomic system *e.g.*, position and momentum; energy and time, angular momentum and angular velocity etc. Quantitatively, this principle states that the order of magnitude of the product of uncertainties in the simultaneous measurements of the two canonically conjugate quantities must be at least of the order of Plank's constant; $\dfrac{h}{2\pi}$

$$\Delta p_x \Delta x \geq \hbar, \quad \Delta J\, \Delta\phi \geq \hbar, \quad \Delta E\, \Delta t \geq \hbar$$

Radius of Bohr's first orbit : According to uncertainty principle, $\Delta x\ \Delta p \approx \hbar$ or $\Delta p \approx \hbar / \Delta x$

$\therefore$ Uncertainty in kinetic energy $\Delta K = \dfrac{(\Delta p)^2}{2m} = \dfrac{1}{2m}\dfrac{\hbar^2}{(\Delta x)^2}$

The potential energy of the electron may be given as $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(-e)}{\Delta x}$ where Z is the atomic number and Ze is the charge on the nucleus.

i.e., maximum uncertainty in the epotential energy can be potential energy itself, *i.e.*,

$$\Delta V_{max} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{\Delta x}$$

Certainty in total energy, $\Delta E = \Delta K + \Delta V_{max} = \frac{h^2}{2m(\Delta x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{\Delta x}$

The minimum value of energy must be at least comparable to the uncertainty in the energy. For minimum uncertainty in energy, one must have

$$\frac{d(\Delta E_{min})}{d(\Delta x)} = 0 \quad or \quad -\frac{h^2}{m(\Delta x)^3} + \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{(\Delta x)^2} = 0$$

or $$\Delta x = 4\pi\varepsilon_0 \frac{h^2}{mZe^2} = \frac{\varepsilon_0 h^2}{\pi m Ze^2}$$

This value of Δx has the same dimensions as the radius of Bohr's first orbit.

Q. 3. (b) What is physical significance of wavefunction ? Derive time independent schrodinger wave equation.

Ans. Physical significance of wave function ψ

$$\psi = A_e^{-i/h}(ET - P_x)$$

Here : $\psi \rightarrow$ Wave function, $P \rightarrow$ Momentum operator, $E \rightarrow$ Energy operator

The Quantity whose variations make up the matter waves is called wave function and it is represented by a break latter Ψ

$\therefore$ $\psi = A + iB$

conjugate of ψ is $\Psi^* = A - iB$ and $\Psi^*\Psi = A^2 + B^2$

$$|\Psi|^2 = A^2 + B^2$$

Now let us assume that a wave function Ψ is specify in x-direction by the wave equation

$$A = Ae^{-i\omega}\left(t - \frac{x}{v}\right) \quad ...(1)$$

where $\omega = 2\pi v$ and $u = v\lambda$

Put in equation (1)

$$\therefore \quad \Psi = Ae^{-2\pi vi}\left(t - \frac{x}{v\lambda}\right) = Ae^{-2\pi i}\left(tv - \frac{x}{\lambda}\right) \quad ...(2)$$

But we know that

$$\therefore \quad E = hv, \quad v = \frac{E}{h}, \quad \lambda = \frac{h}{p} = \frac{2\pi h}{p}, \quad \frac{1}{\lambda} = \frac{p}{2\pi h}$$

Put in equation (1) we get $\Psi = Ae^{-2\pi i}\left(tv - \frac{xp}{2ph}\right) = Ae^{-2\pi c}\left(\frac{tE}{h} - \frac{xp}{2ph}\right)$

$$= Ae^{-2\pi i}\left(\frac{tE}{2\pi h} - \frac{xp}{2ph}\right) = Ae^{-i/h}(Et - Px) \quad (3)$$

Time Independent Schrodinger Wave Equation : It has been found that in some situations. The potential energy of a particle does not depend on time and vary with the position of the particle. When this is true, Schrodinger's equation may be simplified by removing time.

The one dimensional wave function Ψ of an unrestricted particles may be written as follows :

Hence Ψ is the product of a time-dependent function $e^{-(iE/h)t}$ and a position dependent function $\Psi[(\Psi = Ae^{(ip/h)^n}]$.

Differentiating equation (1) with respect to 't', we have

$$\frac{d\psi}{\partial t} = Ae^{-(i/h)(Et - px)} \times \left(\frac{-i}{h} E\right) = \frac{-i}{h} E\psi \qquad ...(2)$$

Also we know that schrodinger time dependent wave equation is,

$$ih \frac{\partial \psi}{\partial t} = \frac{-h^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + v\psi \qquad ...(3)$$

Using equation (1) and (2), Equation (3) becomes

$$E\psi e^{-(iE/h)t} = \frac{-h^2}{2m} e^{-(iE/h)t} \frac{\partial^2 \psi}{\partial x^2} + V\psi e^{-(iE/h)t}$$

Dividing by $e^{(iE/h)t}$, we have

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{h^2}(E - v)\,\psi = 0 \qquad ...(4)$$

In three dimensions,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{h^2}(E - V)\,\psi = 0$$

or
$$\nabla^2 \psi + \frac{2m}{h^2}(E - V) = 0 \qquad ...(5)$$

Equation (4) and (5) are the time independent wave equation for one dimension and three dimension respectively.

Q. 4. Attempt any one part of the following :

Q. 4. (a) What is compton effect ? Derive an expression for the compton shift.

Ans. Compton Shift : When a chromatic beam of high frequency radiation. For example X-rays is scattered by a substance, the scattered radiation contains the radiation of lower frequency or greater wavelength along a radiation of unchange wavelength along a radiation of unchange wavelength in the scattered light is called unmodified radiation while the radiation of greater wave length is called modify radiation. The phenomena is called the Compton effect and the change in $\Delta\lambda$ is called the compton shift.

Loss in photon energy = Gain in electroenergy

$$hv - hv' = K \qquad ...(1)$$

Since, the collision is elastic therefore, total linear momentum of photon and electron before collision is equal to after collision.

Along x-axis : $\dfrac{hv}{c} + 0 = \dfrac{hv'}{c}\cos\phi + P\cos\theta$

$$Pc\cos\theta = hv - hv'\cos\phi \qquad ...(2)$$

Along y-axis : $0 + 0 = \dfrac{hv'}{c}\sin\phi - P\sin\theta$

$$Pc\sin\theta = hv'\sin\phi \qquad ...(3)$$

Squaring and adding equation (2) and (3)

$$P^2c^2(\sin^2\theta + \cos^2\theta) = (hv - hv'\cos\phi)^2 + hv'\sin\phi)^2$$

$$P^2c^2 = h^2v^2 - 2h^2 v v'\cos\phi + h^2v'^2 \qquad ...(4)$$

Also, Initial energy of e^- + Gain in energy = Final energy of electron

$$m_0c^2 + K = \sqrt{(m_0c^2)^2 + P^2c^2}$$

Now, squaring both sides $\quad m_0^2c^4 + K^2 + 2m_0c^2K = m_0^2c^4 + P^2c^2$

Now, comparing it with equation (4)

$$K^2 + 2m_0c^2K = h^2\upsilon^2 - 2h^2\upsilon\upsilon' \cos\phi + h^2\upsilon'^2$$

$\because$
$$K = h\upsilon - h\upsilon'$$

$$(h\upsilon - h\upsilon')^2 + 2m_0c^2(h\upsilon - h\upsilon') = h^2\upsilon^2 - 2h^2\upsilon\upsilon' \cos\phi + h^2\upsilon' - 2h^2\upsilon\upsilon' + 2m_0c^2h(\upsilon - \upsilon')$$

$$= 2h^2\upsilon\upsilon' \cos\phi$$

Now, dividing by $2n^2c^2$

$$-\frac{\upsilon\upsilon'}{c^2} + \frac{m_0}{h}(\upsilon - \upsilon') = \frac{-\upsilon\upsilon'}{c^2}\cos\phi$$

$$\frac{m_0}{h}(\upsilon - \upsilon') = \frac{\upsilon\upsilon'}{c^2}(1 - \cos\phi)$$

$$\frac{m_0c}{h}\left(\frac{\upsilon}{c} - \frac{\upsilon'}{c}\right) = \frac{\upsilon}{c}\cdot\frac{\upsilon'}{c}(1 - \cos\phi)$$

$\because$
$$\frac{\upsilon}{c} = \frac{1}{\lambda} \text{ and } \frac{\upsilon'}{c} = \frac{1}{\lambda'}$$

$$\frac{m_0c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{\lambda\lambda'}(1 - \cos\phi) \qquad \text{...(5)}$$

$$\frac{m_0c}{h}(\lambda' - \lambda) = 1 - \cos\phi$$

$$\lambda' - \lambda = \frac{\lambda}{m_0c}(1 - \cos\phi)$$

Q. 4. (b) Derive claussius - Mossotti Equation in terms of relative permittivity.

Ans. Clausis-Mosotti Relation : Let us take elemental dielectric having cubic structure. As there are no ions and permanent dipoles in these materials, the ionic polarizability α_i and orientational polarizability α_0 are zero.

i.e., $\qquad \alpha_i = \alpha_0 = 0$

Thus polarization $\qquad P = N\alpha_e E_i = N\alpha_e\left(E + \frac{P}{3\varepsilon_0}\right)$

i.e., $\qquad P\left[1 - \frac{N\alpha_e}{3\varepsilon_0}\right] = N\alpha_e E \text{ or } P = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right)} \qquad \text{...(1)}$

We know that, $\qquad D = P + \varepsilon_0 E \quad \text{or} \quad P = D - \varepsilon_0 E$

or $\qquad \frac{P}{E} = \frac{D}{E} - \varepsilon_0 = \varepsilon - \varepsilon_0 \qquad [\because D = \varepsilon_0 E]$

$$= \varepsilon_0\varepsilon_r - \varepsilon_0 \qquad \left[\because \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}\right]$$

or $\qquad P = E\varepsilon_0(\varepsilon_r - 1) \qquad \text{...(2)}$

From equation (1) and (2) we have

$$P = E\varepsilon_0(\varepsilon_r - 1) = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\varepsilon_0}} \quad \text{or} \quad 1 - \frac{N\alpha_e}{3\varepsilon_0} = \frac{N\alpha_e}{\varepsilon_0(\varepsilon_r - 1)}$$

or
$$1 = \frac{N\alpha_e}{\varepsilon_0(\varepsilon_r - 1)} + \frac{N\alpha_e}{3\varepsilon_0} = \frac{N\alpha_e}{3\varepsilon_0}\left(\frac{3}{\varepsilon_r - 1} + 1\right)$$
$$= \frac{N\alpha_e}{3\varepsilon_0}\left(\frac{3 + \varepsilon_r - 1}{\varepsilon_r - 1}\right) = \frac{N\alpha_e}{3\varepsilon_0}\left(\frac{\varepsilon_r + 2}{\varepsilon_r - 1}\right)$$

Hence,
$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0} \quad \text{...(3)}$$

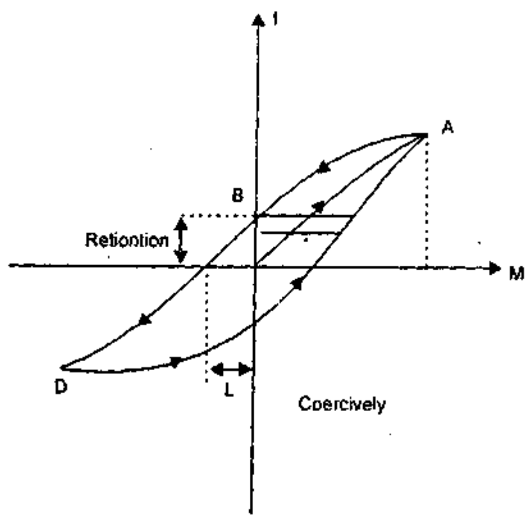
Where N is the number of molecules per unit volume.

Relation (3) is known as clausiues-Mosotti equation. From this relation, we can determine the value of α_e knowing the value of ε_r.

Q. 5. Attempt any one part of the following :

Q. 5. (a) What is hysteresis curve ? Explain residual magnetism, coercive force and hysteresis.

Ans. Hysteresis : The magnetisation curve of a ferromagnetic material is not a linear curve *i.e.*, the variation of magnetic flux B (or I) does not vary linearly with the applied field H. The curve of B (or I) versus H in which the material is magnetised in one direction then in other direction is known as hysteresis curve.



If we vary the magnetising field H, the intensity of magnetisation I of the increases along OA non-uniformly in the substance. At the point A the point A the material acquires a state of magnetic saturation. Further increase in field does not produce any increase in I.

If now the magnetising field H is decreased, the magnetisation J of material also decreases but does not follow the original paths. So J lags luluied H. When H becomes zero I still has a value equal to value OB. This is known as 'Residual magnetism'. The power of retaining this magnetism is called the retentivity of the material so retentivity of the material is a measure of remaining magnetism in the material.

If further now the field is increased in reverse direction the J decreases along BC, still lagging luluied H until it becomes zero at C *i.e.*, equals to OC. The value of OC is called the coercivity of the substance. So coercivity is a measure of reverse magnetising field required to destroy the magnidistant. As H is increased beyond OC the substance is increasingly magnetised in opposite direction. At point D the substance is again magnetised saturated.

By taking H back froms its maximum –ve value (through zero) or its original value a symmetrical curve is obtained. There are two points at which the substance is magnetised in absence of applied field. This is known as permanent magnetism.

Thus the magnetisation I and also the magnetic flux density always lag behind the field. This

Q. 5. (b) What are ultrasonics ? Discuss application of ultrasonics.

Ans. Ultrasonic Waves : We all know that sound is due to vibrations of one or the other kind of particles. The human ear can hear the sound waves between 20 Hz to 20 kHz. This range is called audible range. The sound waves having frequencies above the audible range are called *ultrasonic waves or supersonic waves*. The wavelengths of ultrasonic waves are very small as compared to audible sound. Most of the applications of the ultrasonic waves have been possible on account of their small wavelengths. The sound waves which have frequencies less than the audible range are known as *infrasonic waves*.

Applications of Ultrasonic Waves

(1) Detection of flaws in metals : Ultrasonic waves can be used to detect flaws in metal. We know that flow in the metal gives a change in the medium due to which reflection of ultrasonic waves takes place. Thus when ultrasonic waves pass through a metal having some hole or crack inside it, an appreciable reflection occurs. The reflection also takes place at the back surface of the metal. The reflected pulses are picked up through receiver and are suitably amplified. These pulses are now applied to one set of plates of cathode ray oscillograph. The transmitted signal and reflected signal from the flaw and back surface of metal gives, a peak each. The position of the second perk on the time base of oscillograph will gives distance of flaw.

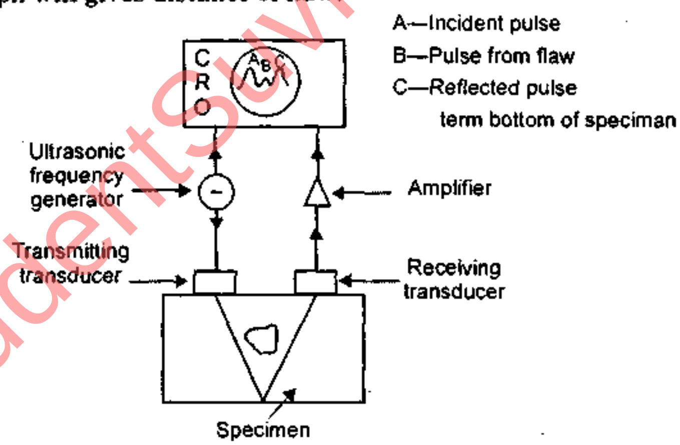


Fig. Ultrasonic flaw detector

The experimental arrangement is given in Fig. Here the transmitting transducer sends a beam of ultrasonics from the material under test. In the presence of flaw in the specimen, the waves will be reflected back and the corresponding recorded intensity in the receiver will be very weak. Similarly, when there is a crack in the specimen, the transmitted waves will have the intensity extremely small. The reflected beam is recorded by using cathode ray oscilloscope.

(2) Sonar : It is possible to determine the presence of submerged submarines or an enemy aircraft by a system called Sonar. Sonar is a device which stands for sound Navigation and Ranging. In this system, a sharp ultrasonic beam is directed in differents directions into the sea. These are picked up on their return after reflection. The reflection of waves from any direction shows the presence of some reflecting body in the sea. The time interval between the generation of ultrasonic waves and their return after reflection gives the idea of the distance of the body. The change in frequency of the echo signal due to Doppler effect helps to find the velocity of the body and its

(3) Soldering and metal cutting : Ultrasonic waves can be used for drilling and cutting processes in metals. These waves can also be used for soldering, *e.g.*, aluminium cannot be soldered by normal methods. To solder aluminium ultrasonic wave along with electrical soldering iron is used. Ultrasonic welding can be done at room temperatures.

(4) Depth of Sea : We know that ultrasonic waves are highly energetic and show a little diffraction effect. Hence they can be used for finding the depth of the sea. The time interval between sending the wave and the reflected wave from the sea is recorded. Since the velocity of the wave is known, thus the depth of the sea can be estimated. Depth of sea $= vt / 2$.

(5) Formation of alloys : The constituents of alloys, having widely different densities, can be kept mixed uniformly by a beam of ultrasonics. Hence it is easy to get alloy of uniform composition.

(6) Direction signalling : The ultrasonic waves can be concentrated into a sharp beam because of smaller wavelength and hence can be used for signalling in a particular direction.

(7) Cleaning and clearing : These waves can be used for cleaning utensils, washing clothes, removing dust and sort from the chimney.

(8) Detection of abnormal growth : Abnormal growth in the brain, certain tumours which cannot be detected by X-rays can be from ultrasonic waves.

(9) Ultrasonics in metallurgy : To irradiate molten metals which are in the process of cooling so as to refine the grain size and to protect the formation of cores and to release trapped gases, the ultrasonic waves are used.

(10) Ultrasonic mixing : A colloid solution or emulsion of two non-miscible liquids like oil and water can be formed through simultaneously subjecting to ultrasonic radiations. Now-a-days most of the emulsions like polishes, paints, food products and pharmaceutical preparations are prepared by using ultrasonic mixing.

Q. 6. Attempt any one part of the following :

Q. 6. (a) Derive Maxwell's equations. Explain the physical significance of each equation.

Ans. Derivative of Maxwell's First Equation : From Gauss's law in electrostatics, we have

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\epsilon_0} \int_V \rho dV \quad \text{or} \quad \oint_S \epsilon_0 \vec{E} \cdot \vec{dS} = \int_V \rho \, dV$$

or $$\oint_S \vec{T} \cdot \vec{dS} = \int_V \rho \, dV \qquad (\because \vec{D} = \epsilon_0 \vec{E})$$

Using Gauss divergence theorem, we have

$$\int_V div\, \vec{D} \cdot dV = \int_V \rho \, dV \quad \text{or} \quad \int_V (div\, \vec{D} - \rho)\, dV = 0$$

or $$div\, \vec{D} - \rho = 0 \quad \text{or} \quad div\, \vec{D} = \rho \qquad \text{...(A)}$$

(2) Derivation of Second Equation : We know that the isolated magnetic poles cannot exist; the magnetic induction across any closed surface is always zero *i.e.*,

$$\oint_S \vec{B} \cdot \vec{dS} = 0$$

Using Gauss divergence theorem, we have

$$\int div\, \vec{B} \cdot dV = 0 \quad \text{or} \quad div\, \vec{B} = 0 \qquad \text{...(B)}$$

(3) **Derivation of Third Equation :** According to Faraday's law of electromagnetic induction, the induced emf is,

$$e = -\frac{\partial \phi}{\partial t}$$

But the magnetic flux ϕ is expressed in terms of magnetic flux density B is

$$\phi = \int_s \vec{B} \cdot d\vec{S}$$

$\therefore$induced emf $\quad e = -\frac{\partial}{\partial t} \int_s \vec{B} \cdot d\vec{S} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot dS$ ...(1)

The emf 'e' may also be expressed in terms of electric field E is, $e = \int_c \vec{E} \cdot \vec{dt}$ ...(2)

From equation (1) and (2),

$$\int_c \vec{E} \cdot \vec{dt} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Using Stoke's theorem,

$$\int_s curl\, \vec{E} \cdot d\vec{S} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

or $$\int_s \left(curl\, \vec{E} + \frac{\partial \vec{B}}{dt} \right) \cdot d\vec{S} = 0$$

or $$curl\, \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

or $$curl\, E = -\frac{\partial \vec{B}}{\partial t}$$ ...(C)

(4) **Derivation of Fourth Equations :** Now curl $H = J + \frac{\partial D}{\partial t}$

From Ampere's circuctial law, $\int_c H \cdot dl = I$ ...(1)

The current I may be expressed in terms of current density J as $I = \int_S J \cdot ds$ ...(2)

From (1) and (2), $\int_c H \cdot dl = \int_s J \cdot ds$

$$\int_c curl\, H \cdot ds = \int_s J \cdot ds$$

(by using stokes theorem to change line integral into surface integral)

$$curl\, H = J$$ ...(3)

Now, divergence of equation $div\ curl\, H = div\ J$

But $div\ curl\, H = 0;\ \ div\ J = 0$ ...(4)

From continuity equation we have $div\ J + \frac{\partial P}{\partial t} = 0$

or $$div\ J = \frac{-\partial P}{\partial t}$$

$\therefore$equation (4) to be valid, $\frac{\partial P}{\partial t}$ should be zero, *i.e.*, the charge should be static.

Hence, to include the time varying fields, Maxwell suggested that Ampere's law must be modified. The current density J should be replaced by $J + J_d$; where J_d is the current density for displacement current.

The equation (3) becomes, $curl\, H = J + J_d$ ...(5)

$$div\, curl\, H = div\, J + J_d \quad \text{(By taking } div\text{)}$$

$$div\, J + div\, J_d = 0$$

$$div\, J_d = -\, div\, J$$

But from continuity equation; $div\, J = -\dfrac{\partial P}{\partial t}$;

Hence, $$div\, J_d = \frac{\partial P}{\partial t}$$

From Gauss's law in differential from, we have $div\, D = P$ ...(6)

Substituting the value of P from equation (6) we get

$$div\, J_d = \frac{\partial}{\partial t}(div\, D) = div\left(\frac{\partial D}{\partial t}\right)$$

This gives $$J_d = \frac{\partial D}{\partial t} \quad \text{...(7)}$$

Substituting the value of J_d from equation (7) in (5), we get $curl\, H = J = \dfrac{\partial D}{\partial t}$ ...(D)

Thus, the Maxwell's fourth equation is the modified from Ampere's law.

Equations (A), (B), (C) and (D) are known as Maxwell's equations.

Physical significance of Maxwell's equations

(a) First equation : This represents that the electric flux linked with a hypothetical closed surface is I / ε_0 times the total charge enclosed by that surface. Thus, this is nothing but the Gauss's law in electrostatics.

(b) Second equation : This equation signifies that the net magnetic flux through any closed magnetic surface is always zero. Thus, any closed surface will always contain the north as well as southpole and the flux entering from one pole will always be equal to the flux leaving to another pole. This indicates that magnetic monopole does not exist. This equation represents the Gauss's law in magnetostatistics.

(c) Third equation : This equation signifies that a changing magnetic flux produces an electricfield. Thus, this is the Faradays law of electromagnetic induction.

(d) Fourth equation : According to this equation, a magnetic field may be produced by a conduction current as well as by changing flux. Thus, this is the modified form of the Ampere's circuital law.

Q. 6. (b) Prove that the velocity of plane electromagnetic wave in the vacuum is given by

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Where various terms have their usual meaning.

Ans. Wave equations in free space : Consider the case of electromagnetic phenomenon in free space or more generally, in a perfect di-electric containing no charge ($p = 0$) and no conduction currents($J = 0$). For this case the field equations become :

$$\nabla \cdot D = 0 \quad \text{...(a)}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{...(b)}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \quad \text{...(c)}$$

$$\partial \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} \quad \text{...(d)}$$

For free space, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 0$ and $\rho = 0$

$$\vec{D} = \varepsilon_0 \vec{E} \text{ and } \vec{B} = \mu_0 \vec{H}$$

Taking curl of equation (c) on both sides, we have

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \nabla \times \frac{\partial \vec{H}}{\partial t} \quad \text{...(1)}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \frac{\partial \vec{H}}{\partial t} = \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{...(2)}$$

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$\therefore$ $$\boxed{\nabla \times (\nabla \times \vec{E}) = \nabla \nabla \cdot \vec{E} - \nabla^2 \vec{E}}$$

and $$\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} \nabla \cdot \vec{D} = 0$$

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{...(3)}$$

Similarly, we can solve for magnetic field $\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \dfrac{\partial^2 \vec{H}}{\partial t^2}$ ...(4)

Equations (3) and (4) represent wave equations.

The waves which serves as a building block in the study of EM waves consist of electric and magnetic fields that are perpendicular to each other and to the direction of propagation and are uniform in planes perpendicular to the direction of propagation. These waves are called uniform plane waves.

Equations (3) and (4) are the wave equations for field vector $\vec{E}$ and $\vec{B}$. As we know the classical equations of wave propagating with velocity υ is given by :

$$\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \quad \text{...(5)}$$

If we compare equations (3), (4) and (5), it follows at once that the field, vectors can be propagated as waves in free space and the velocity of propagation is

$$\frac{1}{\upsilon} = \mu_0 \varepsilon_0 \quad \mu^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} \cong 3 \times 10^8 \text{ m/sec} \equiv C$$

$$\therefore \quad C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

So there exist a electromagnetic wave in space and they travel in free space with velocity of light.

Q. 7. Attempt any one part of the following :

Q. 7. (a) Discuss characteristic properties and uses of superconductors.

Ans. Properties of Superconductors : Following are few properties of super conducting materials :

1. The transition temperature T_c is different for different isotopes of an element. It decreases with increasing atomic weight of the isotopes.

2. At room temperature, super-conducting materials have greater resistivity than other elements.

3. There is no change in the crystal structure as revealed by X-ray diffraction studies. This indicates that superconductivity may be more concerned with the conduction electrons than with the atom itself.

4. The super-conducting property of a super conducting element is not lost by adding impurities to it but the critical temperature is lowered.

5. If a sufficient strong magnetic field is applied to superconductor below critical temperature, its super conducting property is destroyed. At any given temperature below T_c, there is a critical magnetic field H_c such that the super-conducting property is destroyed by the application of magnetic field. The value of H_c decreases with rise in temperature.

6. All thermoelectirc effects disappear in super-conducting state.

7. The thermal expansion and elastic properties do not change in transition.

Applications of Super Conducting Materials

The phenomenon of superconductivity finds many practical applications.

(a) Superconducting coils in transformers and electrical machines generate much stronger magnetic fields than magnetic circuits employing ferromagnetic materials do. Cores made of ferro or ferri magnetic materials will not be needed and eddy current losses and hysteresis losses will be eliminated. Hence, the size of motors and generators will be drastically reduced. They will be lighter and much more efficient. Hence, superconductors are likely to revolutionize the whole range of rotating electrical machines, making them smaller, lighter and highly efficient.

(b) The most spectacular application would be maglev or magnetic levitation trains. Maglev coaches do not slide over steel rails but float on a four inch air crushion over a strongly magnetized track. Superconducting coils produce the magnetic repulsion in order to levitate the coaches. Since there does not exist mechanical friction, speeds upto 500 km/hr can be easily achieved.

(c) The semiconductor logic elements have a speed limit. They works at speeds of nanoseconds. In contrast, logic elements based on Josephson junctions can operate at a speed of a few picoseconds. Josephson junctions are thus expected to increase the speed of supercomputers.

(d) Superconductors will radically change the IC fabrication. At present owing to heat generated through I^2R losses, there is a limit to which the components can be crowded on a chip of

(e) Various medical diagnostic equipments are now employing SQUIDs which detect very minute changes in the magnetic field of a human brain or body.

(f) High magnetic fields are required in many areas of research and diagnostic equipments in medicine. The electromagnets are cumbersome being very big in size, demand large electrical power to maintain the magnetic field and needs continuous cooling. Superconducting solenoids produce very strong magnetic fields. They are small in size and does not need power. Hence, they are less cumbersome and less expensive.

(g) The Meissner effect can be emboided in bearings that would operate without friction losses in all kinds of rotating machines.

(h) Electricity is transmitted and distributed through cables. A large amount of power, about on fifth the quantity generated, is lost on way due to I^2R losses. If superconductors will be used as cables, these losses are avoided and electrical power transmission can be done at a lower voltage level.

Q. 7. (b) How the carbon nanotubes are produced ? Discuss properties and uses of carbon nanotubes.

Physical constants :

Planck's constant $h = 6.62 \times 10^{-34}$ J.S

Mass of neutron $m_n = 1.67 \times 10^{-27}$ kg

Permeability of free space $(\mu_0) = 4\pi \times 10^{-7}$ T.m/A

Permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12}\ c^2 / N.m^2$

Ans. Carbon Nanotube : Carbon nanotubes (CNT) are sheets of graphite (graphite is an allotropic[1] form of pure and brittle form of carbon) rolled up to make a tube it means carbon macro-molecules in cylindrical form. The nanotube dimensions are variable and can be as small as 0.4 nm in diameter. A typical computer generated model of carbon nanotubes is given in figure.

Properties of Carbon Nanotubes

(i) Carbon nanotubes are very hydrophobic and can easily be bind to proteins. Because of this property, they can serve as chemical and biological sensors.

(ii) They also exhibit intersecting electrical properties *i.e.*, depending on the way the graphite structure spirals around the tube, CNTs can be isulating, semiconducting or conducting.

(iii) CNTs are very light, flexible, thermally stable and durable, and possess extraordinary tensile strength. CNTs have tensile strength of about 65 GPa which is 50 times higher than steel.

Structure of Carbon Nanotubes : The bonding in carbon nanotubes is sp^2, with each atom joined to three neighbours similar to those in graphite structure. The tubes can thus be considered as rolled-up graphite sheets. The structure of a nanotube can be specified by a **chiral vector** (n, m) which defines how the graphite sheet is rolled up, where n and m are integers of the vectors equation.

The chiral vector can be understood with the help of Fig. (b). The values of n and m determine the *chirality*, or *twist* of the nanotube. The chirality affects the conductance, density, lattice structure and other properties. A single walled CNT is considered metallic if the value is divisible by 3 otherwise, the nanotube is semiconducting. Consequently, if tubes are formed with random values of n and m we would expect that two-thirds of nanotubes would be semi-conducting, while the rest would be metallic. Given the chiral vector (n, m), the diameter of a carbon nanotube can be determined using the relationship

$$d = (n^2 + m^2 + nm)^{1/2}\ 0.0783 \text{ nm}$$

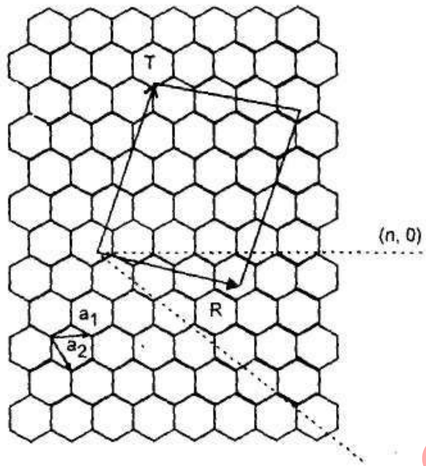


Fig : The (n, m) nanotube can be throught of as a vector in an infinite graphite sheet that describes how to "roll up" the graphite sheet to make the nanotube. T represents the tube axis, and a_1 and a_2 are the unit vectors of graphite.

Types of Carbon Nanotubes : Depending upon the value of n and m, the carbon nanotubes have been divided into following three categories :

(i) If $n = m$, the nanotubes are called "armchair".

(ii) If $m = 0$, the nanotubes are called "zigzag".

(iii) For any other combination of n and m nanotubes are known as "chiral".

Uses of Carbon Nanotubes

1. The carbon nanotube is often used as a vessel for transporting drugs into the body. The nanotube allows for the drug dosage to hopefully be lowered by localizing its distribution, as well as significantly cut costs to pharmaceutical companies and their consumers.

2. Nanotubes based transistors have been made that operate at room temperature.

3. Because of the great mechanical properties of the carbon nanotube, a variety of structures have been proposed ranging from every day items like clothes and sports gear to combat jackets.

4. Bulk carbon nanotubes have already been used as composite fibers in polymers to improve the mechanical, thermal and electrical properties of the bulk product.

5. Some other potential applications have been proposed for carbon nanotubes, including conductive and high-strength composites; energy storage and energy conversion devices; sensors; field emission displays and radiation sources; hydrogen storage media; and nanometer-sized semiconductor devices, probes, and interconnect.

6. Carbon nanotubes have also been implemented in nanoelectromechanical systems, including mechanical memory elements and nanoscale electric motors.